Journal of Computational Acoustics, Vol. 6, Nos. 1 & 2 (1998) 151–166

NEURAL DIRECT APPROACHES FOR GEOACOUSTIC INVERSION

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Received 6 October 1997
Revised 27 November 1997

This paper presents different neural network approaches for geoacoustic inversion. The basic idea of neural inversion is to approximate the inverse function from a set of behaviors, i.e. relations between acoustic fields and geoacoustic parameters. In this work, such methods have been applied in two different forms: a global approach aiming to estimate all parameters from all data, and a hierarchical approach in which the most sensitive parameters are estimated before the least sensitive. The methods are tested using synthetic data. Statistical results, as well as benchmark results, show that such approaches are efficient and have similar performances.

1. Introduction

Numerous studies have been carried out in the recent past to estimate seafloor geoacoustic parameters from acoustic pressure fields recorded using vertical or horizontal arrays [1,2,3]. The motivations for such studies are twofold. On the one hand, seafloor acoustic remote sensing can be seen as an alternative approach to traditional measurements (corings, sediment sonar,...), which generally require time-consuming procedures. On the other hand, acoustic remote sensing methods are adapted to sonar systems for the operational point of view and can be seen as a way to determine an "equivalent" bottom consistent with what sonar systems see.

Different methods have been developed in the purpose of geoacoustic inversion. Most popular methods rely on a global or a local search in the parameter space. Several drawbacks may be exhibited: such methods are generally time-consuming and have to be run for each new set of data. In addition, estimating the accuracy of the solution (error bars) is difficult. Neural networks, considered as tools of nonlinear regression, may efficiently avoid these drawbacks.

Recently, the use of neural networks was proposed for bottom and ocean tomography[4,5].

Neural nets have proved to be efficient nonlinear approxi-mators[6]. The basic idea of neural inversion is to approximate the inverse function from a set of behaviors, i.e. the relations between acoustic fields and geoacoustic parameters. In this work, such methods have been applied in two different forms: a global approach aiming to estimate all parameters from all data, and a hierarchical approach in which the most sensitive parameters are estimated before the least sensitive.

The paper is organized as follows: Section 2 recalls the forward problem and presents a sensitivity study. Section 3 presents the inverse methodology based on the use of neural networks. Section 4 gives statistical results of the methods and the results on the benchmark cases.

2. Forward problem and sensitivity study

It is well known that the acoustic pressure measured in the water column strongly depends on the sound reflection from the interfaces. In particular, the attenuation is weaker when the grazing angle is small and when the number of bottom reflections is few. As a consequence, the pressure field at long ranges in shallow water (typically several kilometers) mainly involves acoustic energy emitted at low grazing angles. The corollary of this is that contributions from high grazing angles is noticable at short ranges only. Furthermore, the energy emitted with high grazing angles goes deeper in the bottom and is more sensitive to substratum features than the energy emitted with low grazing angles.

In the frequency domain, it is also well known that high frequencies are more attenuated than low frequencies. One can therefore expect that low frequency energy is more informative about the basement. Such qualitative assessments suggest that a trade-off can be found to retrieve satisfactory information about the bottom from a "cleverly" selected set of data. One of the objectives of the sensitivity study is to provide information to select such a data set.

2.1. *Sensitivity study*

The sensitivity study aims to numerically compare the influence of each geacoustic model parameter on the data with respect to depth, range and frequency. The aim is to rank all model parameters from the most sensitive to the least sensitive. This ranking allows to predict which parameters might be accurately estimated. It also allows to design a hierarchical inversion scheme. For practical reasons that will be explained hereafter, we have focused on the influence of range and frequency. With this aim, we have designed a particular measure of sensitivity. To estimate the influence of a given model parameter, we discretize its range of variation in N values. For each value, the transmission loss (in dB) is computed with all other parameters frozen at their mean value. The mean absolute value of the difference between transmission losses for consecutive values of the model parameter is computed (Figure 1).

Then, the mean vertical transmission loss is calculated and plotted versus range. This measure is thus the mean variation of the mean transmission loss on the water column, when the studied parameter changes by a $(N-1)$ part of its total range of variation. Therefore,

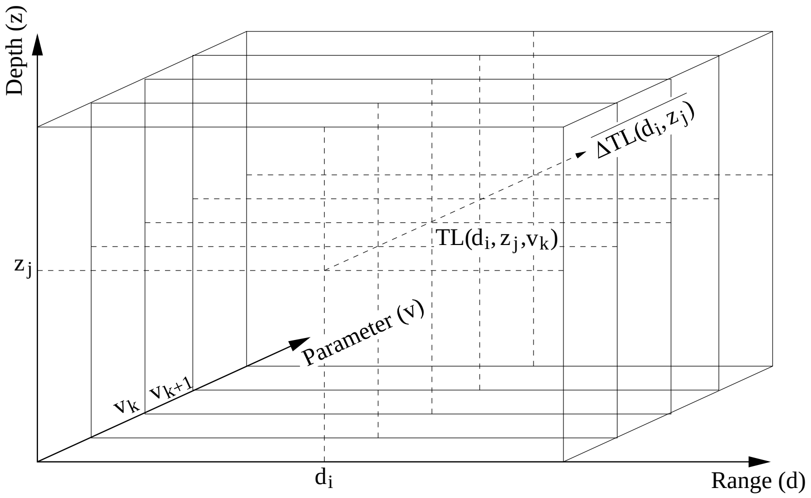

Fig. 1. Principle of the sensitivity measure (TL: Transmission Loss).

the values of the mean variation, given in dB, can be compared provided that the number N of discrete values is the same for all parameters. Nevertheless, it must be kept in mind that this method is numerical and mainly depends on the range of variation of the parameters. That is, a very influential parameter restricted to a short range of variation may be found to be less important than another parameter given a wide range of variation.

The sensitivity of all parameters, including the source-receiver configuration, is estimated for three frequency scenarios (in each scenario, the number of values for each parameter is $N = 23$): mid frequency (400 Hz), broadband (300-500 Hz), and very low frequency (25 Hz). The search ranges are described in the benchmark cases[7] (the AT case search intervals are also described in Table 1). The results of the sensitivity study are given in Figure 2 (400 Hz), Figure 3 (broadband) and Figure 4 (25 Hz). Several conclusions can be drawn.

(i) The influence of incoherent broadband processing is a smoothing of the sensitivity function.

(ii) At a given frequency (or range of frequencies in the broadband case), the sensitivity of each parameter varies with range.

(iii) At mid frequency, the predominant parameters are non geoacoustic parameters, the most influential being the water depth. Concerning geoacoustic parameters, the sediment is more influential than the basement, as expected. The influence of most parameters decreases with range, the sediment parameters remaining the only sensible ones at long ranges ($>$ 2 km). The basement influence can only be seen through its compressional speed at short ranges ($<$ 2 km).

(iv) At low frequency, water depth is still the predominant parameter. The influence of source-receiver parameters is weakened due to the fact that the wavelength is significantly longer at 25 Hz than at 400 Hz. Sediment and basement parameters are all influental, at a comparable rate. The influence of all parameters varies with range on a periodic cycle length of 700-800 m, with an increasing trend.

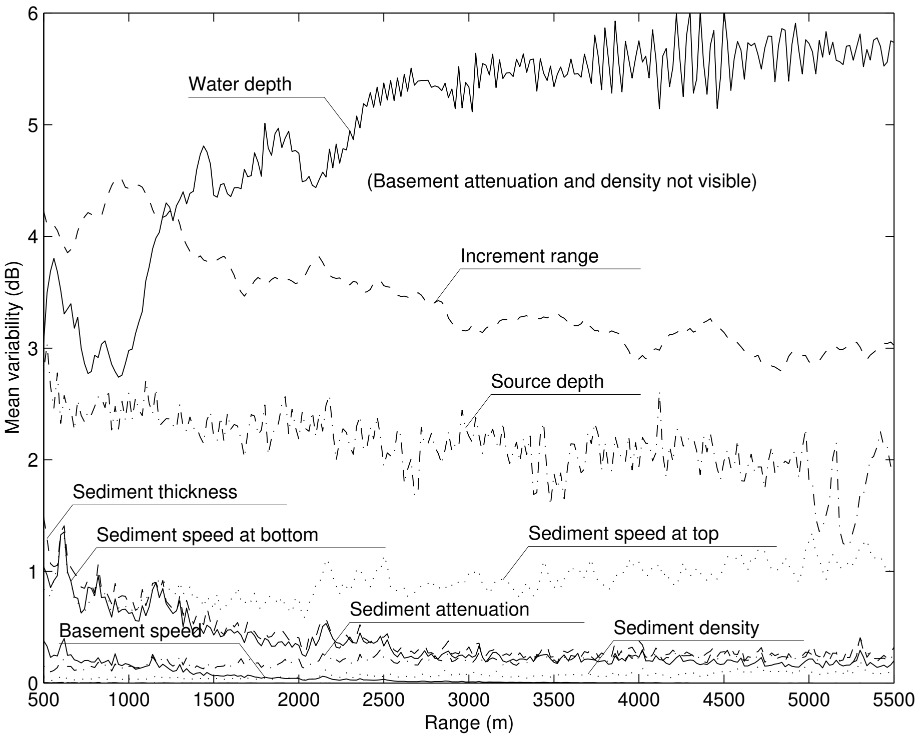

Fig. 2. Results of the sensitivity study (400 Hz).

2.2. *Selection of data*

The last step is to efficiently choose the set of data to be inverted. Two points have to be considered. On one hand, we aim to select the most informative data. On the other hand, we have to stay within practical constraints (particularly in terms of instrumention). In this framework, we have chosen to work with a realistic configuration similar to that of the INTIMATE'98 experiment[8] (to be carried out in July 1998). We have excluded very low frequency (the broadband source will work between 300 and 1,000 Hz) and the vertical array will have 12 hydrophones equally spaced in the water column between 10 and 65 meters. From the previous analysis, the vertical array has to be placed in the near field to be sensitive to basement parameters. Nevertheless, data collected at long ranges can also be of use because the upper sediment speed continues to be a sensitive geoacoustic parameter. In summary, all inversions presented below come from the following data processing configuration:

- 12-hydrophone vertical array between 10 and 65 meters,
- transmission losses at 1 km and 5 km,
- range of frequency 300-500 Hz,
- frequency incoherent processing.

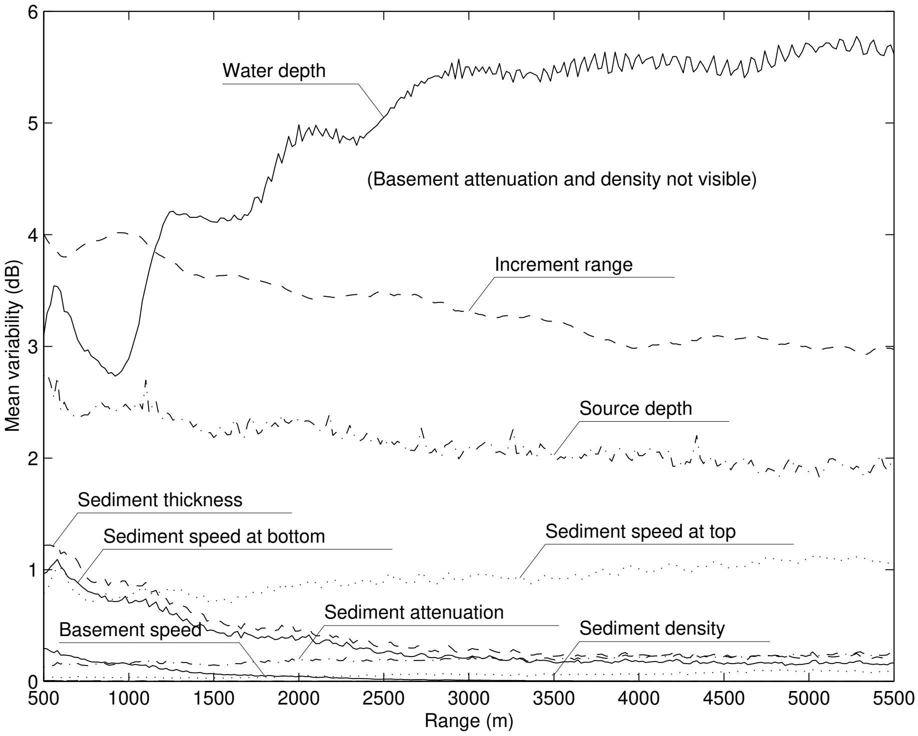

Fig. 3. Results of the sensitivity study (broadband 300-500 Hz).

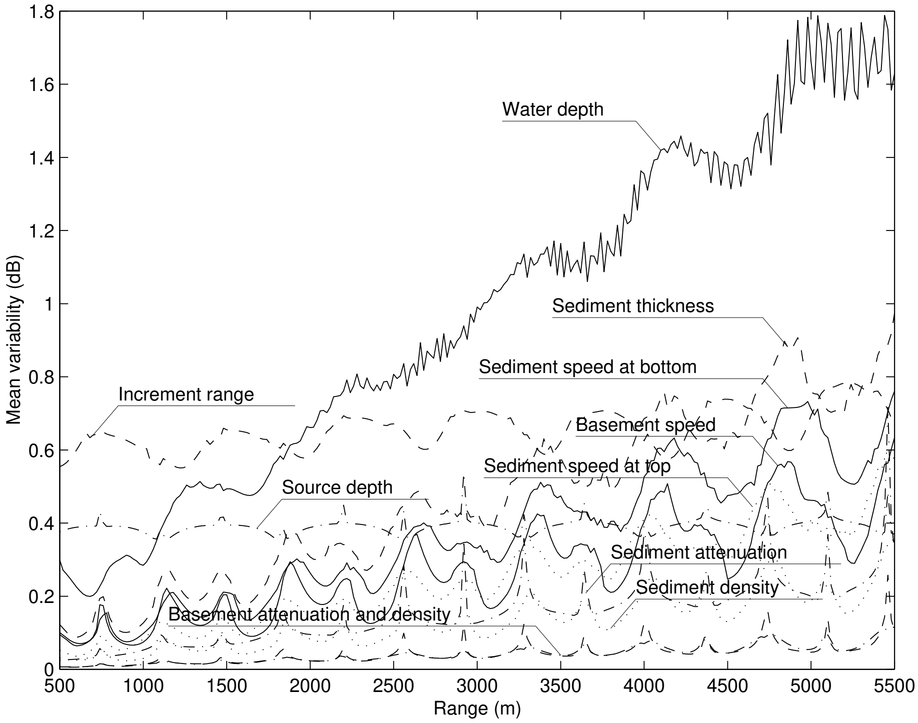

Fig. 4. Results of the sensitivity study (25 Hz).

3. Inversion

3.1. *Neural networks for inversion*

The main advantage of neural networks is their ability to adapt themselves to perform a predefined task. In more mathematical terms, neural networks are universal (nonlinear) function approximators. As such, they can be used in two different ways for inversion[9]:

- a direct approach, consisting in directly approximating the inverse model from a given set of behaviors for that model,
- a variational approach, by approximating both the forward and adjoint models.

In this paper, we will focus on the first approach only.

3.1.1. *Multilayer Perceptrons*

A neural network is a set of interconnected elementary processors (so-called neurons or cells), each one performing a simple function f (the transfer function) of the weighted sum of its inputs. The output y_j of neuron j is computed following equation :

$$y_j = f(a_j) \quad where \quad a_j = \sum_i w_{ij} x_i. \tag{3.1}$$

The transfer function of elementary units may take different forms. The most usual are the sigmoidal and identity function.

Numerous neural networks exist, but the best suited for function approximation are multilayer Perceptrons (hereafter MLP). In MLP, neurons are connected in layers (Figure 5). One layer is receiving the system inputs and is called the input layer. One layer is providing the network outputs and is called the output layer. One or more layers are internal to the network and are called the hidden layers. The layers are fully connected (each neuron in a layer is connected to every neuron in the previous layer) and the information propagates from the input to the output (no backward propagation). Biases (additional weights connected to a constant input) can be added on the output nodes and on the hidden nodes. In this work, the transfer functions of hidden units are sigmoidal and the transfer functions of input and output units are linear.

The problem of multidimensional approximation can be posed as follows[10]. Denoting by $\Re$ the space of real numbers, let F be a real function defined by:

$$\begin{gathered} F : \Re^Q \to \Re^P \\ \mathbf{x} = (x_1, ..., x_Q)^T \to \mathbf{y} = (y_1, ..., y_P)^T = F(\mathbf{x}). \end{gathered} \tag{3.2}$$

where $\Re^Q$ (respectively $\Re^P$) denotes the cartesian product $\Re \times \Re \times ... \times \Re$ Q times (respectively P times). We assume that F is prescribed by a set $\mathcal{L}$ of N behaviors $(\mathbf{x}t^i, \mathbf{y}t^i), i = 1, ..., N$. The task is to find an approximation function $G(W, \mathbf{x})$ which best matches F on $\mathcal{L}$. The matrix W is the matrix of the internal parameters of the approximator G. The approximation is achieved by finding a configuration W^* such that, for any given W:

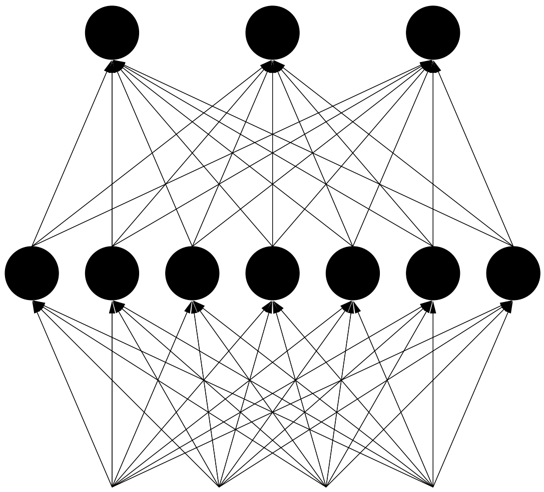

Fig. 5. Structure of a multilayer Perceptron.

$$\|G(W^*, \mathbf{x}t) - F(\mathbf{x}t)\| \leq \|G(W, \mathbf{x}t) - F(\mathbf{x}t)\|. \tag{3.3}$$

The notation $\|...\|$ stands for an arbitrary norm in $\Re^P$. This definition is general and is not specially related to the neural net concept. However, it can be easily interpreted in the neural framework: W is the matrix of the weights, and the approximation function $G(W, \mathbf{x})$ generalizes the function F from a set of "learned" examples $\mathcal{L}$ ($\mathcal{L}$ is called the learning base). For this approximation task, the MLP is built as follows. Each $\mathbf{x}t$-component is encoded on one neuron of the input layer. Each $\mathbf{y}t$-component is encoded on one neuron of the output layer. The misfit between the target values and the actual outputs of the net is measured by a quadratic cost function:

$$E(W) = \frac{1}{2} \sum_{m=1}^{N} \sum_{j=1}^{P} \left[y_j(W, \mathbf{x}t^m) - yt_j^m \right]^2. \tag{3.4}$$

The optimal W configuration is found by minimizing the cost function using for instance a steepest gradient descent method (gradient backpropagation algorithm[11]). After each presentation of an input-output pattern (step s), the weight between neuron k and neuron l is modified following the rule:

$$w_{kl}^s = w_{kl}^{s-1} - \alpha \left[\frac{\partial E}{\partial w_{kl}} \right]^s, \tag{3.5}$$

where α is a small positive real number (often called learning gain).

The ability of MLP to approximate continuous multidimensional function defined on compact subsets has been proved theoretically[12,13]. Given a function F, there exists a multilayered neural network with one hidden layer (but more than one can be used) able to approximate F in the uniformly convergent sense. Nevertheless, there are no absolute rules to determine the optimal architecture of the network. It must be done heuristically.

Approximating a function by increasing the number of hidden units (on a single or on several hidden layers) is always possible. However, to be efficient, the approximation has to be done on a number of examples "much greater" than the number of weights. Generally, arbitrarily increasing the number of hidden units (i.e the number of weights in the net) leads to poor performances in generalization.

3.1.2. *Direct inversion*

In the direct approach, the inverse model is approximated from a set of behavior. The underlying idea is to build a numerical inverse model (the MLP) able to estimate some model parameters from measurements (data) in two steps (Figure 6):

(i) Learning of the inverse model with an MLP.
(ii) Estimating the model parameters by presenting new observations to the network.

This method is quite easy to use, and very fast once the network has been configured.

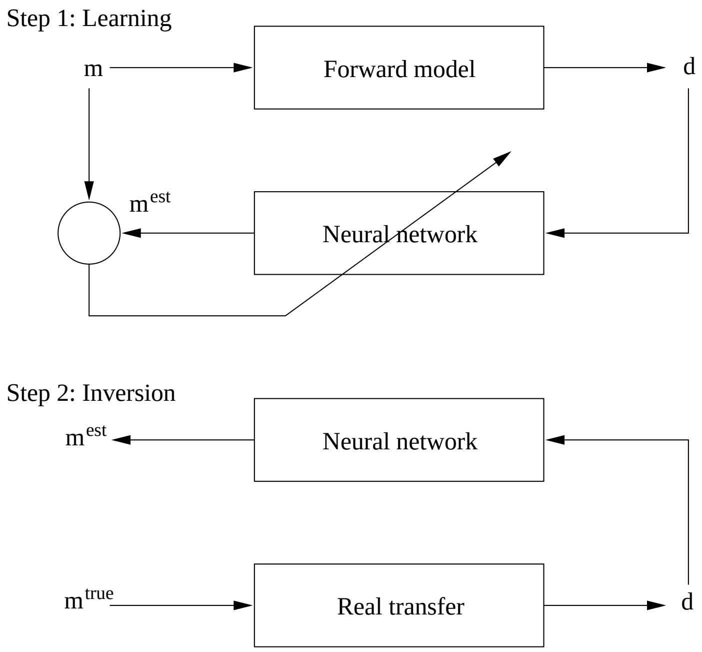

Fig. 6. Principle of the neural direct inversion (d: data, m: model parameters).

3.2. *Application to geoacoustic inversion*

The application of the method to geoacoustic inversion is straightforward. The data are the transmission losses on a vertical array and the model parameters are the geoacoustic and source-receiver configuration parameters.

To have a good representation of the phenomena, i.e. of the contribution of each parameter to the acoustic field, a representative sampling of the parameter space is required. The more influential the parameter is, the finer the sampling must be. Table 1 presents an

	Parameter	*LB*	*UB*	*Step*	*Number*
Sediment	speed at top (m/s)	1500	1600	5	21
	speed at bottom (m/s)	1550	1750	10	21
	thickness (m)	10	50	5	9
	attenuation (dB/λ)	0.05	0.5	0.03	16
	density (g/cm^3)	1.4	1.7	0.1	4
Basement	speed (m/s)	1600	1800	20	11
	attenuation (dB/λ)	0.05	0.5	0.2	3
	density (g/cm^3)	1.6	1.8	0.2	3
Total number of configurations					2.5×10^7

Table 1: A sampling of the search space. LB and UB stand for Lower Bound and Upper Bound.

example of the minimal sampling needed to obtain a rough estimation of the geoacoustic parameters for the AT case. This sampling produces 2.5×10^7 possible environments to be tested. Even with the fastest computers and models, this is not feasible.

This is why most techniques of inversion try to "find a way" in the search space that reaches the solution (or at least a good solution) without exploring the whole search space. The neural network approach is quite different, relying instead on nonlinear regression. An optimal black-box model of the inverse function is built from a set of behavior. The search space has to be well sampled in the learning base. After learning, the search space is not used any more. The inversion is done instantaneously, the network performing a nonlinear regression between the examples that were chosen for the learning. When no prior knowledge is available, the best way to build the learning base is to uniformally sample the search space between the allowed bounds. For each benchmark case, 1,000 examples were generated for learning using the OASES code[14]. To test the performances of the inverse method, 400 additional examples were also considered (test base).

3.2.1. *Global approach*

The global approach is straightforward. A network is built to retrieve the estimate of all model parameters from all data (Figure 7).

3.2.2. *Hierarchical approach*

The hierarchical approach relies on the sensitivity study, which shows that some parameters are more influential than others at certain ranges. For example, Figure 3 shows that the sediment speed at top is the only sensitive geoacoustic parameter at long range (5 km). Thus, a good approximation of this parameter should be obtained from the 5-km data. At short range (1 km), three parameters dominate: the sediment speed at top and bottom, and sediment thickness. Thus, the upper sediment speed having previously been inverted from the 5-km data, 1-km data should allow retrieval of the two other parameters. The three

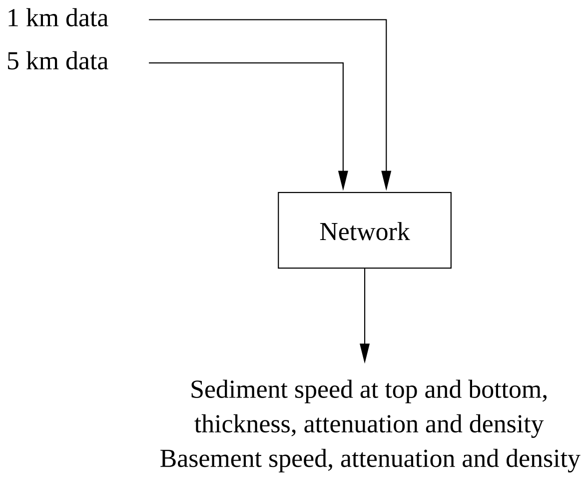

Fig. 7. Principle of the neural global inversion (AT case).

previous parameters being known, the sediment density and attenuation may best be found from 5-km data. Finally, knowing all the sediment parameters, the basement parameters may best be found from the 1-km data.

The input of the network has to be chosen carefully. If only the transmission losses are given, networks will become specialized networks that together perform a simple global inversion. To take advantage of the hierarchical ranking, networks have to be fed with the transmission losses and the parameters which are more influential than those the network estimates. For example, once the upper sediment speed has been inverted from 5-km data, the next network must be fed with 1-km data and at least the upper sediment speed to retrieve the sediment thickness and lower sediment speed. The other parameters can also be added. This last scheme seems more interesting because it allows feedbacks between all networks. In this case, the process is iterative. On the other hand, it also suggests that the initialization of the solution might predetermine the outcome. Both inversion schemes are summarized in Figure 8.

4. Results

4.1. *Statistical results*

Statistical results on the AT test cases for the global inversion, and both forms of hierarchical inversion (one-step and iterative) are discussed next. The error is the ratio between the absolute error and the variation range of each parameter. This definition allows a fair comparison for methods when the solution is searched in a given interval. Its drawback is that "good" intervals (*a priori information*) have to be defined. In this study, intervals are described in the benchmark test-cases[7].

Table 2 gives a comparison between different methods for the AT case. Several conclusions can be drawn.

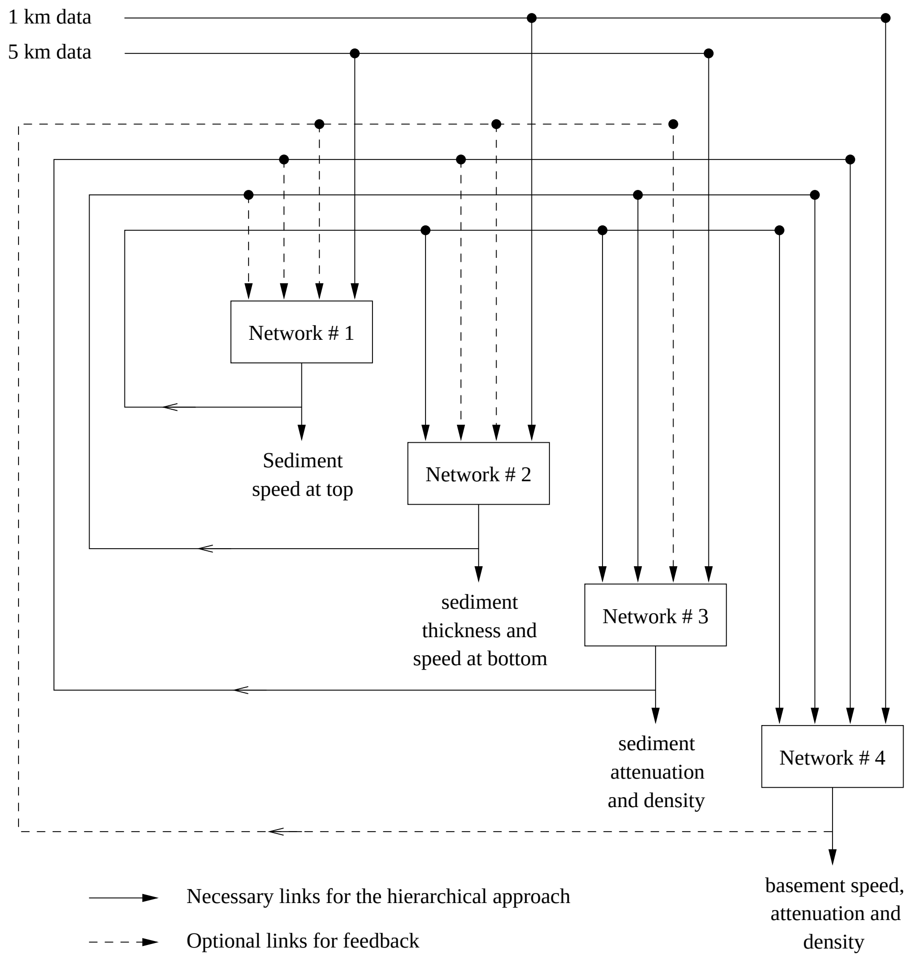

Fig. 8. Principle of the neural hierarchical inversion (AT case).

(i) Estimation of model parameters: As expected from the sensitivity study, sediment parameters are better estimated than basement parameters. The ranking of the parameters, in terms of inversion accuracy, is the following: upper sediment speed, attenuation, density, lower sediment speed and thickness, basement speed, attenuation and density. This ranking is in accordance with the sensitivity study except for the sediment thickness and lower sediment speed. In fact this may not be contradictory because the sensitivity study also shows that these two parameters are correlated.* This is noticable in Figure 3 where the two curves are almost merged and have the same trends even for small scale variations.

(ii) Comparison between methods: There are no significant differences between methods. In particular, there is no evidence of improvement by using a hierarchical approach. This may be due to the fact that the estimation error on each parameter is propagat-

*The correlation between the lower sediment speed and sediment thickness was widely discussed during the benchmark workshop held in Vancouver in June 1997. See more details in this issue of JCA.

Parameter		*Mean error (%)*					
		Learning base			*Test base*		
		A	*B*	*C*	*A*	*B*	*C*
Sediment	speed at top	4.1	**3.6**	4.0	**4.1**	4.2	4.1
	speed at bottom	**15.6**	22.9	20.4	**20.7**	24.2	22.9
	thickness	16.3	18.7	**16.0**	19.2	20.2	**17.8**
	attenuation	**5.0**	6.4	5.6	**5.2**	6.7	5.8
	density	**10.5**	13.6	13.6	14.8	**14.8**	15.0
Basement	speed	**17.2**	22.8	18.5	19.7	23.4	**19.6**
	attenuation	**23.1**	25.3	23.2	**24.8**	25.5	26.4
	density	23.1	24.8	**22.9**	**25.6**	26.8	26.0

Table 2: Comparison of the performances of the methods on the learning and test bases for the AT case. A: global inversion, B: one-step hierarchical inversion, C: iterative hierarchical inversion. Best results are in bold. The error is the ratio between the absolute error and the variation range of each parameter (see Table 1 in this case).

ing from network to network. This has been confirmed by running the hierarchical approach with true values for the unsearched parameters. For instance, if network 2 of Figure 8 is fed with 1-km data and the true value of the upper sediment speed, then the mean error for the lower sediment speed and thickness is 16.9 % and 14.6 %, instead of 24.2 % and 20.2 %, respectively.

4.2. *Benchmark results*

Considering the above statistical results, benchmark data are inverted with the global neural approach, since it is easier to use than hierarchical approaches and has similar results.

To compute error bars, a gaussian perturbation with 5 % standard deviation is added on data. One hundred realizations of the perturbated data are inverted. The *a posteriori* errors are computed from the statistical analysis of the estimated parameters. It should be emphasized that these error bars quantify the uncertainty due to data.

Table 3 to 5 respectively present the results of benchmark inversion on the AT, SO and WA cases.

It must be emphasized that the methods presented in this work rely on nonlinear regression theory. Therefore, statistical results as presented for the AT case are more representative of the method ability than inversion on a small set of examples. Nevertheless, the inversions of the benchmark cases are consistent with our expectation. Sediment geoacoustic parameters are accurately estimated (see the results for upper sediment speed and attenuation for the AT case). Basement parameters, less influental, may be well or badly estimated (see for instance the density in the AT case). This is due to uncertainties in inverse model approximation (insensitive parameters remain unavoidably ambiguous).

When source-receiver parameters are included as unknowns to be determined, the performances of the methods significantly decrease (see the upper sediment speed in the SO

Parameter			*Real*	*Estimate*	*Error bar*	*Error (%)*
Sediment	speed at top (m/s)	a	1547	1544	33	2.48
		b	1558	1551	33	7.66
		c	1534	1535	31	0.85
	speed at bottom (m/s)	a	1625	1592	121	16.22
		b	1565	1567	106	0.99
		c	1558	1571	109	6.54
	thickness (m)	a	44.6	37.1	25.4	18.58
		b	13.4	30.2	20.2	41.99
		c	30.4	31.7	20.0	3.23
	attenuation (dB/λ)	a	0.12	0.11	0.23	1.39
		b	0.30	0.26	0.20	9.53
		c	0.08	0.05	0.23	6.67
	density (g/cm^3)	a	1.40	1.40	0.36	0.00
		b	1.77	1.70	0.36	23.33
		c	1.43	1.50	0.29	22.83
Basement	speed (m/s)	a	1676	1689	69	6.19
		b	1759	1693	68	32.91
		c	1615	1668	83	26.30
	attenuation (dB/λ)	a	0.08	0.25	0.14	37.84
		b	0.20	0.26	0.15	14.22
		c	0.05	0.35	0.21	66.44
	density (g/cm^3)	a	1.78	1.76	0.15	4.32
		b	1.95	1.87	0.15	21.01
		c	1.69	1.74	0.15	11.90

Table 3: Results on AT cases.

and WA cases). However, the results remain satisfactory. At any rate, the estimations of the less sensitive parameters are not degraded (see density in the basement for all cases).

5. Conclusion

In this paper, three neural network approaches have been presented to perform the inversion of acoustic data for benchmark test cases. The global method is the simplest: a single network associates the data (transmission losses) to model parameters (geacoustic parameters and source-receiver parameters). The one-step and iterative hierarchical methods rely on a sensitivity study, which has shown some parameters to be more influential than others at certain ranges. In these hierarchical methods, several networks cooperate, each one being specialized in the inversion of a few parameters only. It turns out that all methods give similarly accurate results on average. Good accuracy in retrieving geoacoustic parameters and source location were obtained.

The data used in the inversion were selected in accordance with a detailed sensitivity study. Results could be improved by selecting the appropriate source-receiver separation and

Parameter			*Real*	*Estimate*	*Error bar*	*Error (%)*
Position	distance (km)	a	-0.38	-0.29	0.29	22.02
		b	-0.18	-0.13	0.19	12.50
		c	-0.04	-0.14	0.20	23.79
	source depth (m)	a	25.1	26.1	8.8	5.14
		b	15.3	14.2	5.2	5.63
		c	27.9	30.0	11.3	10.25
Sediment	speed at top (m/s)	a	1593	1576	37	16.33
		b	1510	1506	28	4.16
		c	1542	1552	38	9.78
	speed at bottom (m/s)	a	1699	1706	121	3.61
		b	1745	1683	68	31.44
		c	1674	1677	76	1.23
	thickness (m)	a	22.3	26.5	18.2	10.44
		b	36.0	31.0	18.8	12.59
		c	46.9	27.8	21.6	47.76
	density (g/cm^3)	a	1.61	1.61	0.12	1.06
		b	1.47	1.55	0.15	27.47
		c	1.50	1.60	0.17	33.39
Basement	speed (m/s)	a	1744	1773	114	14.64
		b	1769	1745	63	11.97
		c	1764	1748	71	8.35
	density (g/cm^3)	a	1.80	1.79	0.11	1.97
		b	1.91	1.83	0.14	20.57
		c	1.87	1.87	0.21	1.20

Table 4: Results on SO cases.

frequency, depending on what geoacoustic parameters were of interest. The work presented in this paper has been restricted to the instrumental configuration to be deployed during the INTIMATE'98 experiment, July 98, on the Armorican continental shelf.

Our current work is concerned with the use of neural networks in variational approaches, to reduce ambiguities resulting from the weak sensitivity of some geoacoustic parameters which make the problem ill-posed. Preliminary results (not shown here) indicate significant improvements over direct approaches.

Parameter			*Real*	*Estimate*	*Error bar*	*Error (%)*
Water	depth	a	115	117	8	6.61
	(m)	b	107	105	11	7.61
		c	120	116	9	20.05
Position	distance	a	-0.22	-0.25	0.16	7.42
	(km)	b	-0.10	-0.14	0.19	9.14
		c	-0.29	-0.08	0.17	51.72
	source depth	a	26.4	22.9	10.9	17.73
	(m)	b	26.5	20.4	10.8	30.71
		c	28.2	30.0	12.8	8.85
Sediment	speed at top	a	1516	1525	31	8.47
	(m/s)	b	1584	1595	47	10.70
		c	1566	1539	38	26.83
	speed at bottom	a	1573	1604	93	15.59
	(m/s)	b	1723	1713	102	5.04
		c	1592	1631	88	19.67
	thickness	a	27.1	29.0	15.5	4.82
	(m)	b	31.8	16.3	22.3	38.75
		c	28.9	28.9	15.3	0.13
	density	a	1.54	1.58	0.15	12.03
	(g/cm^3)	b	1.80	1.61	0.21	63.61
		c	1.68	1.55	0.13	43.73
Basement	speed	a	1751	1688	77	31.70
	(m/s)	b	1779	1753	94	13.12
		c	1707	1701	76	3.30
	density	a	1.85	1.83	0.15	4.68
	(g/cm^3)	b	1.86	1.81	0.18	12.07
		c	1.88	1.82	0.14	14.66

Table 5: Results on WA cases.

Aknowledgements

The authors wish to thank A. Tolstoy and R. Chapman for organizing the geoacoustic benchmark workshop. Thanks also to all participants for fruitful discussions. This work was funded by *Service Hydrographique et Océanographique de la Marine (SHOM)* under contract 97.87.011.

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